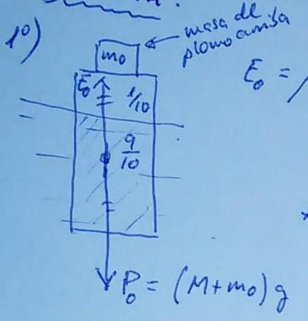
Un bloque de madera de 1 kg de masa y densidad $0{,}7\ g/cm^3$ se deja flotar en el agua cargándole un plomo para que el bloque solo sobresalga del agua $\frac{1}{10}$ de su volumen. ¿Cuál es la diferencia en masa del plomo según se coloque éste encima o debajo?

Solución:

1º)



$$\underbrace{E_0}_{} = \underbrace{\rho}_{\text{densidad del agua}} \cdot 0{,}9 \cdot \underbrace{V}_{\text{volumen madera}} \cdot g = \rho \cdot 0{,}9 \cdot \frac{M}{\rho_m} \cdot g \quad \text{(ppio. Arquímedes)}$$

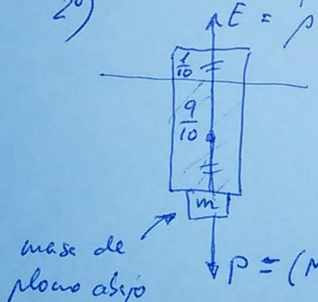
$0{,}9 \cdot V$: volumen desalojado; M: masa madera $= 1\,kg = 1000\,g$; ρ_m: densidad madera

* Equilibrio estático: $P_0 = E_0 \therefore (M+m_0)\cancel{g} = \rho\, 0{,}9 \frac{M}{\rho_m} \cdot \cancel{g} \therefore$

$$\therefore \boxed{m_0 = \left(0{,}9 \frac{\rho}{\rho_m} - 1\right) M} = \left(0{,}9 \frac{1\,\cancel{g/cm^3}}{0{,}7\,\cancel{g/cm^3}} - 1\right) \cdot 1000\,g$$

$$= \left(\frac{9}{7} - 1\right) 1000 = \frac{9-7}{7} \cdot 1000 = \underline{\underline{\frac{2000}{7}\,g}}$$

2º)



$$E = \overbrace{\underbrace{\rho \cdot 0{,}9 \cdot V}_{\text{volumen desalojado madera}} \cdot g}^{\text{madera}} + \overbrace{\underbrace{V_{Pb}}_{\text{volumen desalojado plomo}} \cdot \rho \cdot g}^{\text{plomo}} = \rho \cdot 0{,}9 \cdot \frac{M}{\rho_m} g + \frac{m}{\rho_{Pb}} \cdot \rho g =$$

ρ_{Pb}: densidad del plomo

$$= \rho g \left(\frac{0{,}9M}{\rho_m} + \frac{m}{\rho_{Pb}}\right)$$

* Equilibrio estático: $E = P$

$$\rho \cancel{g} \left(\frac{0{,}9M}{\rho_m} + \frac{m}{\rho_{Pb}}\right) = (M+m)\cancel{g} \therefore$$

$$\therefore 0{,}9M \frac{\rho}{\rho_m} - M = m - m \frac{\rho}{\rho_{Pb}} \therefore \boxed{m = \frac{M\left(0{,}9 \frac{\rho}{\rho_m} - 1\right)}{1 - \frac{\rho}{\rho_{Pb}}}}$$

Dividiendo miembro a miembro las dos expresiones recuadradas:

$$\frac{m}{m_0} = \frac{\frac{\cancel{\left(0{,}9 \frac{\rho}{\rho_m} - 1\right)M}}{1 - \frac{\rho}{\rho_{Pb}}}}{\cancel{\left(0{,}9 \frac{\rho}{\rho_m} - 1\right)M}} = 1 - \frac{\rho}{\rho_{Pb}} = \frac{\rho_{Pb}}{\rho_{Pb} - \rho} \therefore m = m_0 \frac{\rho_{Pb}}{\rho_{Pb} - \rho} \therefore$$

$\therefore$ Diferencia de masa: $m - m_0 = m_0 \frac{\rho_{Pb}}{\rho_{Pb} - \rho} - m_0 =$

$$= m_0 \left(\frac{\rho_{Pb}}{\rho_{Pb} - \rho} - 1\right) = m_0 \frac{\cancel{\rho_{Pb}} - \cancel{\rho_{Pb}} + \rho}{\rho_{Pb} - \rho} \therefore \text{Con } \rho_{Pb} = 11{,}4\ g/cm^3$$

$$\Rightarrow m - m_0 = m_0 \frac{\rho}{\rho_{Pb} - \rho} = \frac{2000}{7} \cdot \frac{1\,\cancel{g/cm^3}}{(11{,}4 - 1)\,\cancel{g/cm^3}} \Rightarrow \boxed{m - m_0 = 27{,}47\,g}$$